

MATH 103 Pre-Calculus Mathematics
Quiz #1 Fall 2008
Sample Solutions

For problems 1 and 2, express the given statement using inequalities.

1. x is in the interval $[-2, 5)$.

Solution: $-2 \leq x < 5$

2. x is in the interval $(-\infty, \sqrt{5}]$.

Solution: $x \leq \sqrt{5}$

For problems 3 and 4, express the given statement using intervals.

3. $3 \leq x$

Solution: x is in $[3, \infty)$

4. $\frac{3}{2} < x \leq \pi$ or $x > 9$

Solution: x is in $(\frac{3}{2}, \pi] \cup (9, \infty)$

5. What is the distance between -7 and -2 ?

Solution: 5

In gory detail, we derive this result as follows

$$\begin{aligned} d(-7, -2) &= |-7 - (-2)| && \text{(Distance formula (page 10))} \\ &= |-7 + 2| && \text{(algebra : } a - (-b) = a + b) \\ &= |-5| && \text{(arithmetic)} \\ &= 5 && \text{(definition of absolute value (page 10))} \end{aligned}$$

For problems 6 through 8 use interval notation to indicate the values of x satisfying the given inequality. Show your work.

6. $x^2 + x \geq 2$

Solution: (Note that \equiv means the same thing as “if and only if”.)

$$\begin{aligned} &x^2 + x \geq 2 \\ \equiv &x^2 + x - 2 \geq 0 && \text{(algebra)} \\ \equiv &(x + 2)(x - 1) \geq 0 && \text{(factoring)} \end{aligned}$$

The product $(x + 2)(x - 1)$ is zero precisely when either $x = -2$ or $x = 1$, which leads us to consider the three intervals $(-\infty, -2)$, $(-2, 1)$, and $(1, \infty)$ (corresponding to the inequalities $x < -2$, $-2 < x < 1$, and $1 < x$, respectively).

To determine the sign of the product $(x + 2)(x - 1)$ in each of these intervals, it suffices to determine the sign of each of the two factors and to apply the rule that the product of two factors is negative if and only if exactly one of the factors is negative. (More generally, the sign of a product is negative if and only if an odd number of its factors are negative.)

This leads to the following table, which indicates the sign of each factor (and, in the last line, the sign of the product) within each interval.

	$(-\infty, -2)$	$(-2, 1)$	$(1, \infty)$
$x + 2$	-	+	+
$x - 1$	-	-	+
$(x + 2)(x - 1)$	+	-	+

From the table, we conclude that the inequality $(x + 2)(x - 1) \geq 0$ holds if and only if x is in either $(-\infty, -2]$ or $[1, \infty)$, which is to say that x is in their union, $(-\infty, -2] \cup [1, \infty)$.

7. $\frac{(2-x)(x+1)}{x-4} < 0$

Solution: One can rewrite the left-hand side of the given inequality as

$$(2 - x)(x + 1) \left(\frac{1}{x - 4} \right)$$

This product is zero or undefined precisely when x is one of -1 , 2 , or 4 . Hence, we consider the four intervals $(-\infty, -1)$, $(-1, 2)$, $(2, 4)$, and $(4, \infty)$ (corresponding to the inequalities $x < -1$, $-1 < x < 2$, $2 < x < 4$, and $4 < x$, respectively).

	$(-\infty, -1)$	$(-1, 2)$	$(2, 4)$	$(4, \infty)$
$2 - x$	+	+	-	-
$x + 1$	-	+	+	+
$1/(x - 4)$	-	-	-	+
$(2 - x)(x + 1)(1/(x - 4))$	+	-	+	-

According to the last row of the table, the given inequality holds precisely when x is in $(-1, 2) \cup (4, \infty)$.

8. $|2x - 1| < 7$

Solution:

$$\begin{aligned} & |2x - 1| < 7 \\ \equiv & -7 < 2x - 1 < 7 \quad (\text{3rd absolute value property, page 11}) \\ \equiv & -6 < 2x < 8 \quad (\text{algebra}) \\ \equiv & -3 < x < 4 \quad (\text{algebra}) \end{aligned}$$

In interval notation, this can be stated as x is in $(-3, 4)$.