

**MATH 103 Pre-Calculus Mathematics**  
**Quiz #4 Fall 2008**  
**Sample Solutions**

1. Give the slope-intercept equation of the line passing through the points  $(1, -3)$  and  $(5, 3)$ . In the space below, sketch the line.

**Solution:** First we ascertain the line's slope:

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - 3}{1 - 5} = \frac{-6}{-4} = \frac{3}{2}$$

Having the line's slope, as well as two of its points (we now need only one of them), we can use the **point-slope** equation form in order to derive the slope-intercept equation:

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{(point - slope form)} \\ y - 3 &= \frac{3}{2}(x - 5) && \text{(plugging in 5, 3, and } \frac{3}{2} \text{ for } x_1, y_1, m) \\ y &= \frac{3}{2}x - \frac{15}{2} + 3 && \text{(arithmetic, algebra)} \\ y &= \frac{3}{2}x - \frac{9}{2} && \text{(arithmetic)} \end{aligned}$$

As an alternative to using the point-slope equation (as we did above), we could have recognized that, for some  $b$  yet to be determined, the line we seek is described by the equation  $y = \frac{3}{2}x + b$ , and, in particular, this equation is satisfied by taking  $x$  and  $y$  to be  $x_1$  and  $y_1$ , respectively, for any point  $(x_1, y_1)$  on the line.

$$\begin{aligned} y_1 &= \frac{3}{2}x_1 + b \\ 3 &= \left(\frac{3}{2}\right)(5) + b && \text{(plugging in 5, 3 for } x_1, y_1) \\ 3 &= \frac{15}{2} + b && \text{(arithmetic)} \\ b &= 3 - \frac{15}{2} && \text{(algebra)} \\ b &= -\frac{9}{2} && \text{(arithmetic)} \end{aligned}$$

Having solved for  $b$ , we get that the point-slope form is  $y = \frac{3}{2}x - \frac{9}{2}$ , which agrees with the solution we obtained before. See graph in Figure 1.

2. Give the slope-intercept equation of the line that passes through the point  $(-2, 3)$  and is perpendicular to the line given by the equation  $x - 2y = 4$ . Sketch both lines in the space below.

**Solution:** Subtracting 4 from both sides of the given equation yields  $x - 2y - 4 = 0$ , which is of the form  $Ax + By + C = 0$ . Recall that the slope of the corresponding line is  $-\frac{A}{B}$ , which in this case (with  $A = 1$  and  $B = -2$ ) is  $\frac{1}{2}$ .

Had you not remembered this technique, you could have taken the given equation and, by subtracting 4 from each side, adding  $2y$  to each side, and then dividing each side by 2, obtained

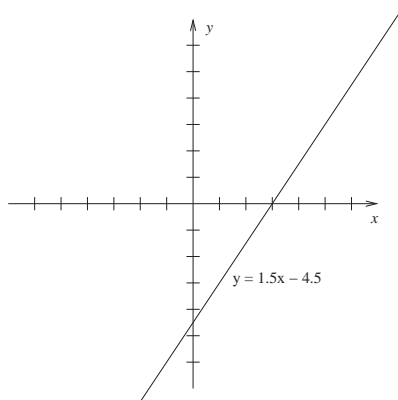


Figure 1: Answer to (1)

the equivalent equation  $y = \frac{1}{2}x - 2$ . This, of course, is in the slope-intercept format and reveals that the line has a slope of  $\frac{1}{2}$ .

Having ascertained that  $\frac{1}{2}$  is the slope of a line that is perpendicular to the line that we seek, we conclude that the line that we seek has slope  $-2$ . (After all, the product of the slopes of any two perpendicular lines (neither of which is vertical) is  $-1$ , and  $\frac{1}{2} \cdot -2 = -1$ .)

In addition to knowing one of its points, namely  $(-2, 3)$ , now we also know the slope of the line that we seek, namely  $-2$ . Beginning with the point-slope equation, we derive the slope-intercept equation:

$$\begin{aligned}
 y - y_1 &= m(x - x_1) && \text{(point - slope form)} \\
 y - 3 &= -2(x - -2) && \text{(plugging in } -2, 3, -2 \text{ for } x_1, y_1, m) \\
 y - 3 &= -2(x + 2) && \text{(arithmetic)} \\
 y &= -2x - 4 + 3 && \text{(arithmetic, algebra)} \\
 y &= -2x - 1 && \text{(arithmetic)}
 \end{aligned}$$

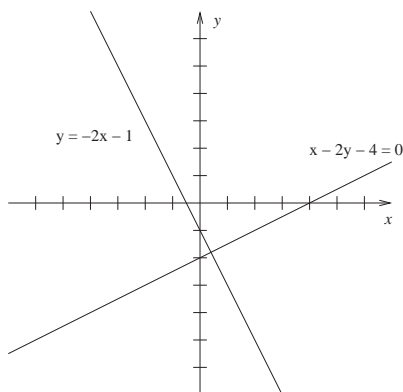


Figure 2: Answer to (2)

**3.** In the space below, sketch the parabola described by the equation  $y = x^2 + 2x + 3$ . Label the vertex by its coordinates.

**Solution:** We use the completing-the-square method to transform the given equation into one of the form  $y = a(x - h)^2 + k$ , which describes a parabola with vertex  $(h, k)$ .

$$\begin{aligned}y &= x^2 + 2x + 3 && \text{(given)} \\y &= (x^2 + 2x + 1) - 1 + 3 && \text{(add 1 and subtract 1)} \\y &= (x + 1)^2 + 2 && \text{(factoring and arithmetic)}\end{aligned}$$

We conclude that the parabola has vertex  $(-1, 2)$  and opens upward. Indeed, the graph is just that of  $y = x^2$  shifted one unit to the left and two units up.

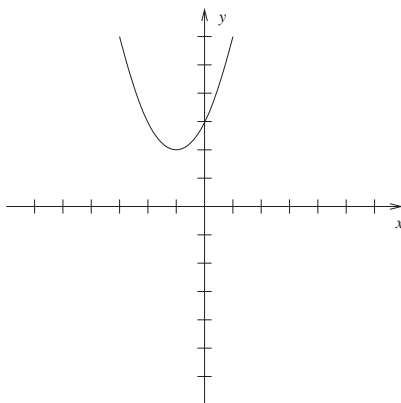


Figure 3: Answer to (3)