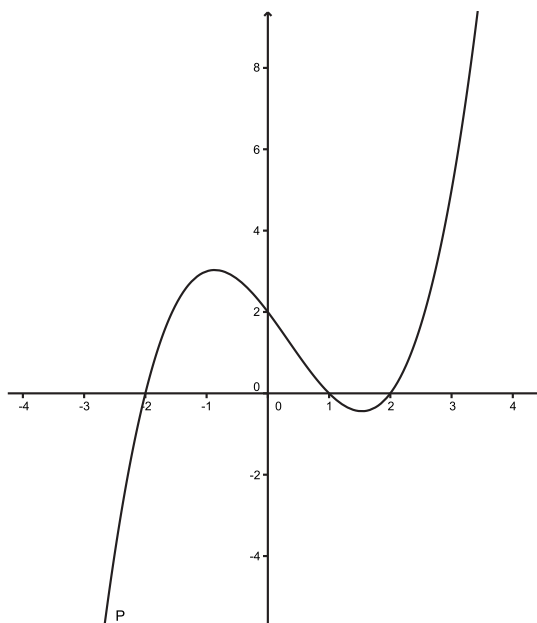


MATH 103 Pre-Calculus Mathematics
Quiz #7 Fall 2008
Sample Solutions

1. Polynomial $P(x)$ has degree 3 and has zeros at $x = 1$, $x = 2$, and $x = -2$. It also includes the point $(0, 2)$. Determine $P(x)$ and sketch its graph.

Solution:



Because P has zeros at 1, 2, and -2 , the Factor Theorem tells us that each of $x - 1$, $x - 2$, and $x + 2$ is a factor of P . That is, for some positive integers i , j , and k and some polynomial Q , we have

$$P(x) = (x - 1)^i(x - 2)^j(x + 2)^k \cdot Q(x)$$

The fact that P has degree 3, however, implies that each of i , j , and k must be 1 and that $Q(x)$ must be a constant. Why? Because, otherwise, P would have degree at least four. Hence, we conclude that, for some constant a , $P(x) = a(x - 1)(x - 2)(x + 2)$. To determine a , we make use of the fact that $P(0) = 2$. Plugging in 0 for x , we get

$$P(0) = a(-1)(-2)(2) = 4a$$

So we have $P(0) = 2$ and $P(0) = 4a$, which means that $2 = 4a$, or $a = \frac{1}{2}$. It follows that

$$P(x) = \frac{1}{2}(x - 1)(x - 2)(x + 2) = \frac{x^3}{2} - \frac{x^2}{2} - 2x + 2$$

2. Match the equation with the graph.

Solution:

(a) $y = (x - 1)(x + 2)^2(x - 2)$ Figure 1

(b) $y = (x - 1)(x + 2)^3(x - 2)$ Figure 3

(c) $y = -x(x + 3)(x - 2)$ Figure 4

(d) $y = -x(x + 3)^2(x - 2)$ Figure 2

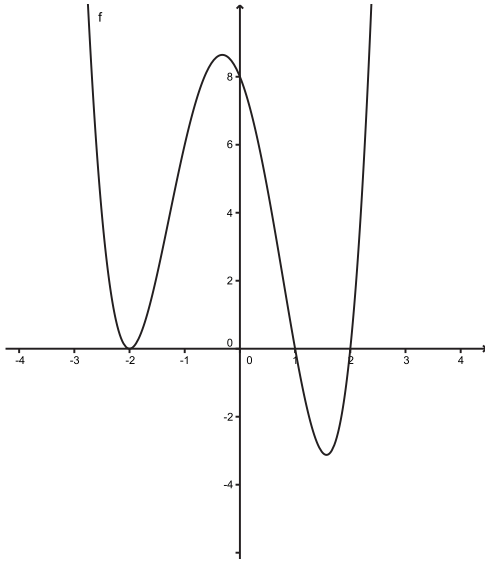


Figure 1:

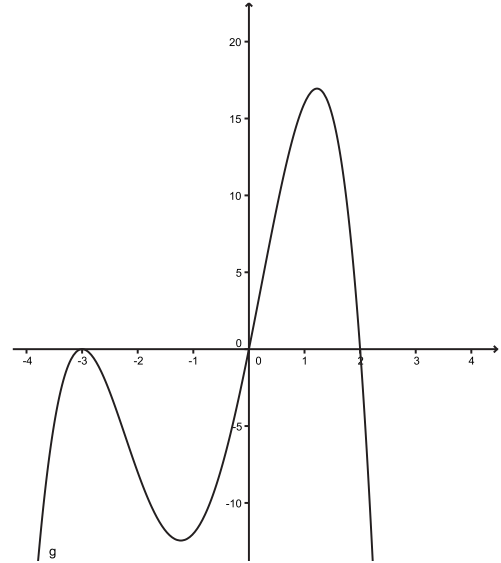


Figure 2:

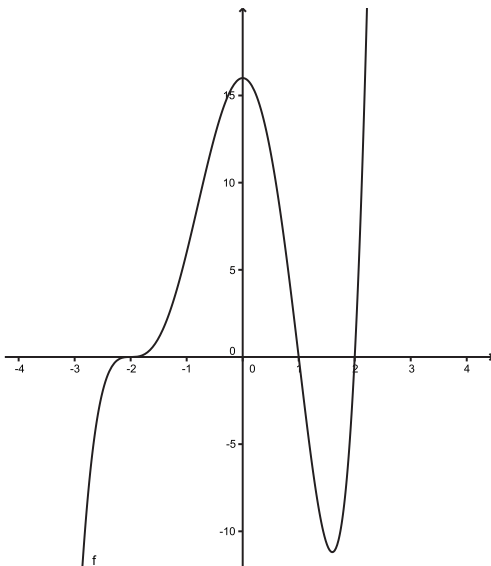


Figure 3:

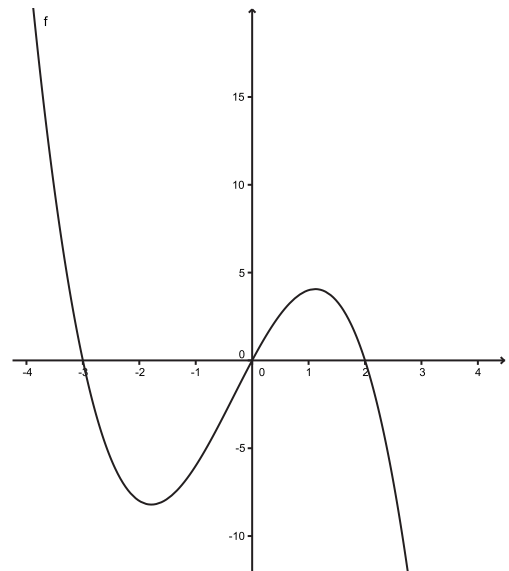


Figure 4: