

Show any relevant work used in arriving at (or verifying) an answer, but be sure to clearly mark that answer.

1. The table below defines the functions f and g , each of which has as its domain the set of integers $\{0, 1, 2, 3, 4\}$. Fill in the missing entries in the table, using **UND** to mean “undefined”. (For example, in the first row you are to supply the values $(f + g)(0)$, $(f - g)(0)$, etc.)

Also answer the questions underneath the table.

x	$f(x)$	$g(x)$	$(f + g)(x)$	$(f - g)(x)$	$(f \cdot g)(x)$	$(f/g)(x)$	$(f \circ g)(x)$	$(g \circ f)(x)$
0	1	4						
1	-1	0						
2	0	1						
3	3	8						
4	2	1						

What is the domain of $(g \circ f)$? What is its range?

2. Suppose that the graph of $y = g(x)$ is obtained from that of $y = f(x)$ by vertically elongating the latter by a factor of 3 and then shifting it four units to the left and seven units down. Describe function g in terms of function f .

3. Below are four graphs of $y = f(x)$, labeled Figure 1 through Figure 4.

In Figure 1, sketch the graph of $y = g(x) = |f(x)|$.

In Figure 2, sketch the graph of $y = g(x) = -\frac{1}{2}f(x)$.

In Figure 3, sketch the graph of $y = g(x) = \frac{1}{f(x)}$.

In Figure 4, relabel the tick marks on the x -axis to obtain a graph of $y = g(x) = f(2x)$.

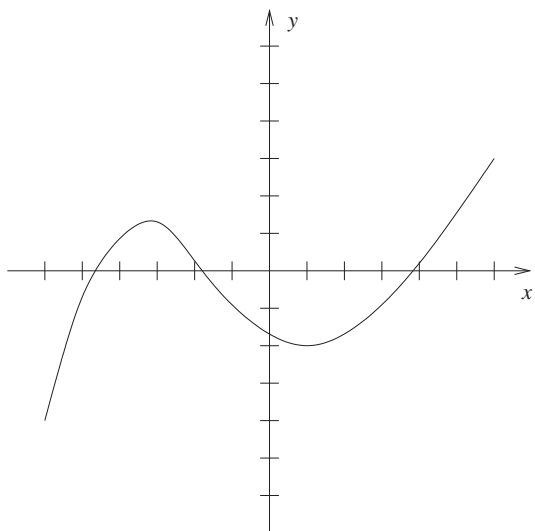


Figure 1: $y = f(x)$ and $y = g(x) = |f(x)|$

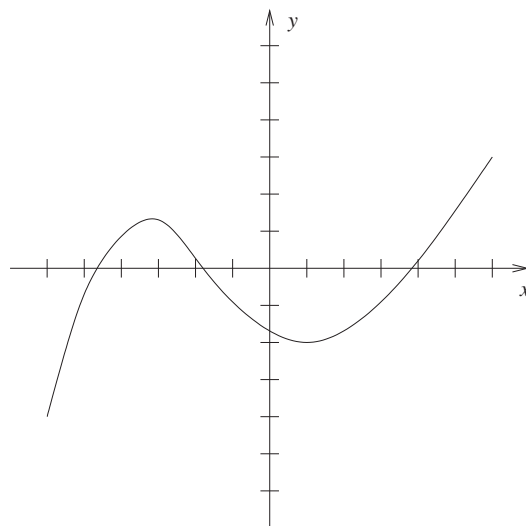


Figure 2: $y = f(x)$ and $y = g(x) = -\frac{1}{2}f(x)$

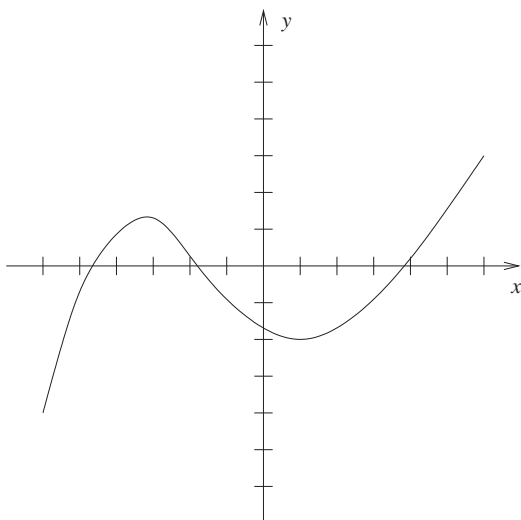


Figure 3: $y = f(x)$ and $y = g(x) = \frac{1}{f(x)}$

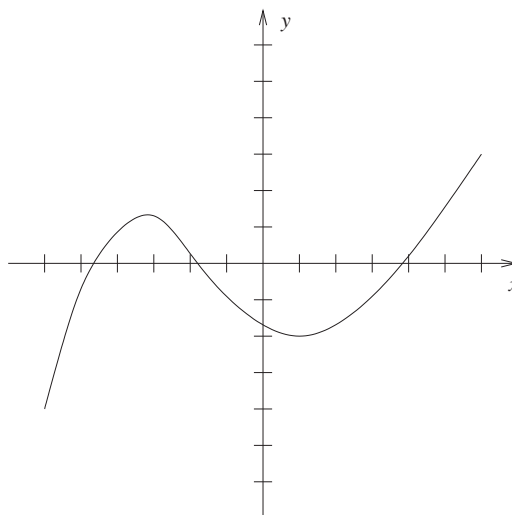


Figure 4: $y = g(x) = f(2x)$

4. For each function f , express it as a composition of two or more of the functions $g(x) = 1/x$, $u(x) = 2x$, $v(x) = x + 3$, $w(x) = \sqrt{x}$, and $h_n(x) = x^n$ (where n is any positive integer). For example, if $f(x) = 2 \cdot (x + 3)^5$, then $f = (u \circ h_5 \circ v)$. Verify at least one of your answers and, preferably, more of them.

(a) $f(x) = x^2 + 3$

(b) $f(x) = (x + 3)^2$

(c) $f(x) = (2x + 3)^4$

(d) $f(x) = \sqrt{2/(x + 3)}$

(e) $f(x) = 4x$

5. For each function, either give evidence that it is not one-to-one or else find its inverse.

(a) $f(x) = -2x + 1$

(b) $f(x) = |x - 1| + 5$

(c) $f(x) = 2\sqrt[3]{x+4} - 1$

(d) $f(x) = x^2 + 2x - 2$

6. Let $f(x) = \frac{1}{(x-1)(x+1)}$ and $g(x) = \sqrt{x}$.

(a) What is the domain of f ?

(b) What is the domain of g ?

(c) What is the domain of $(g \circ f)$?

(d) What is the domain of $(f \circ g)$?

7. (Bonus Problem)

Suppose that f and g are one-to-one functions. Express $(f \circ g)^{-1}$ (i.e., the inverse of $(f \circ g)$) as the composition of two functions. (Hint: both f and g are one-to-one and hence both have inverses.)