**Limit Laws:** If $c$ is a constant and the limits $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then

1. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a} c \cdot f(x) = c \cdot \lim_{x \to a} f(x)$
4. $\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a} [f(x)/g(x)] = \lim_{x \to a} f(x)/\lim_{x \to a} g(x)$ if $\lim_{x \to a} g(x) \neq 0$
6. $\lim_{x \to a} [f(x)]^n = [\lim_{x \to a} f(x)]^n$ for a positive integer $n$
7. $\lim_{x \to a} c = c$
8. $\lim_{x \to a} x = a$
9. $\lim_{x \to a} x^n = a^n$ for a positive integer $n$
10. $\lim_{x \to a} \sqrt{x} = \sqrt{a}$
11. $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$

**Theorem 2.3.0:** If, on some interval $(b, c)$ that includes $a$, $f(x) = g(x)$ except possibly at $x = a$, then

$$\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$$

(assuming that either limit exists).

In part, what this theorem says is that $\lim_{x \to a} f(x)$ does not depend upon $f(a)$ (or whether it is even defined).

**Theorem 2.3.1:**

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^{-}} f(x) = L \text{ and } \lim_{x \to a^{+}} f(x) = L$$

In other words, the (two-sided) limit exists if and only if both of the one-sided limits exist and agree with each other.

**Theorem 2.3.2:** Let $(b, c)$ be an interval including $a$ in which, for all $x$ (except possibly $a$), $f(x) \leq g(x)$. Then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

assuming that both limits exist.
Theorem 2.3.3 (Squeeze/Sandwich/Pinching): Let \((b, c)\) be an interval including \(a\) in which, for all \(x\) (except possibly \(a\)), \(f(x) \leq g(x) \leq h(x)\). Also assume that
\[
\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)
\]
Then
\[
\lim_{x \to a} g(x) = L
\]

Definition 2.4.2 (Limit): Let \(f\) be a function defined on some open interval that contains \(a\), except possibly at \(a\) itself. Then
\[
\lim_{x \to a} f(x) = L
\]
means that, for every number \(\epsilon > 0\) (no matter how small) there exists a number \(\delta > 0\) such that if \(x\) lies in the interval \((a - \delta, a + \delta)\) (but excluding \(a\) itself) then \(f(x)\) lies in the interval \((L - \epsilon, L + \epsilon)\).

Definition 2.4.3 (Left-hand Limit): Let \(f\) be a function defined on some open interval that contains \(a\), except possibly at \(a\) itself. Then
\[
\lim_{x \to a^-} f(x) = L
\]
means that, for every number \(\epsilon > 0\) (no matter how small) there exists a number \(\delta > 0\) such that if \(x\) lies in the interval \((a - \delta, a)\) then \(f(x)\) lies in the interval \((L - \epsilon, L + \epsilon)\).

Definition 2.4.4 (Right-hand Limit): Let \(f\) be a function defined on some open interval that contains \(a\), except possibly at \(a\) itself. Then
\[
\lim_{x \to a^+} f(x) = L
\]
means that, for every number \(\epsilon > 0\) (no matter how small) there exists a number \(\delta > 0\) such that if \(x\) lies in the interval \((a, a + \delta)\) then \(f(x)\) lies in the interval \((L - \epsilon, L + \epsilon)\).

Definition 2.4.6 (Infinite Limit): Let \(f\) be defined on some open interval that contains the number \(a\), except possibly at \(a\) itself. Then
\[
\lim_{x \to a} f(x) = +\infty
\]
means that for every positive number \(M\) there exists a positive number \(\delta\) such that if \(x\) lies in the interval \((a - \delta, a + \delta)\), then \(f(x) > M\).

Definition 2.4.7 (Infinite Limit (negative)): Let \(f\) be defined on some open interval that contains the number \(a\), except possibly at \(a\) itself. Then
\[
\lim_{x \to a} f(x) = -\infty
\]
means that for every negative number \(N\) there exists a positive number \(\delta\) such that if \(x\) lies in the interval \((a - \delta, a + \delta)\), then \(f(x) < N\).

Definition 2.5.1 (Continuity at a point): A function \(f\) is continuous at \(a\) if
\[
\lim_{x \to a} f(x) = f(a)
\]
**Definition 2.5.3:** $f$ is **continuous on an interval** if it is continuous at every number in the interval.

**Theorem 2.5.4** (preservation of continuity by arithmetic operations): If $f$ and $g$ are continuous at $a$, then so are the functions $f + g$, $f - g$, $cf$ (where $c$ is a constant), $fg$, and $f/g$ (if $g(a) \neq 0$)

**Theorem 2.5.5** Each polynomial is continuous everywhere; each rational function (i.e., one of the form $p/q$ where $p$ and $q$ are polynomials) is continuous on its domain.

**Theorem 2.5.7** Each function of any of the following types is continuous on its domain: polynomial, rational function, root function (i.e., of form $\sqrt[n]{x}$), and trigonometric (e.g., $\sin x$, $\cos x$, etc.)

**Theorem 2.5.8** If $f$ is continuous at $b$ and \( \lim_{x \to a} g(x) = b \), then

$$\lim_{x \to a} f(g(x)) = f(\lim_{x \to a} g(x))$$

**Theorem 2.5.9** If $g$ is continuous at $a$ and $f$ is continuous at $g(a)$, then $f \circ g$ is continuous at $a$. In other words, if

$$\lim_{x \to a} g(x) = g(a) \quad \text{and} \quad \lim_{x \to g(a)} f(x) = f(g(a))$$

then

$$\lim_{x \to a} (f \circ g)(x) = (f \circ g)(a) = f(g(a))$$

**Theorem 2.5.10** (The Intermediate Value Theorem (IMVT)): Let $f$ be continuous on the closed interval $[a, b]$, where $f(a) \neq f(b)$ and let $N$ be any number between $f(a)$ and $f(b)$. (That is, either $f(a) < N < f(b)$ or $f(a) > N > f(b)$.) Then there exists $c \in (a, b)$ such that $f(c) = N$. 

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