For each of Problems 1 through 7, indicate the weakness/strength relationship that exists between the two given predicates, $P$ and $Q$. Recall that there are four possibilities: $P$ and $Q$ are equivalent, $P$ is strictly stronger than $Q$, $P$ is strictly weaker than $Q$, or none of the above. For definitions of these, and instructions for how to demonstrate them, follow the On the Strength/Weakness Relationship between Predicates link on the course webpage.

You must justify your answers, but you need not provide formal justifications for “obvious” theorems of arithmetic, such as $x > y \Rightarrow x \geq y$ or $x \geq y + 4 \Rightarrow x \geq y$ or $x < y \Rightarrow x \neq y$.

1. $P : x > 2 \land y > x - 1$ and $Q : x \geq 4 \land y \geq x$
2. $P : x > 2 \land y \geq x - 1$ and $Q : x \geq 4 \lor y > x$
3. $P : x > 2 \lor y < x - 1$ and $Q : x > -5$
4. $P : x > 1 \land y < x$ and $Q : x > -5$
5. $P : x > -5 \land y < x$ and $Q : x = 0$
6. $P : x \geq 0 \Rightarrow y > z$ and $Q : x = 4 \Rightarrow y \geq z$
7. $P : f.k = 5$ and $Q : (\exists i : f.i = 5)$

For the next two problems, use the Strengthening the Precondition, Weakening the Postcondition, Precondition Disjunctivity, and Postcondition Conjunctivity Laws of Hoare Triples (a link to which you can find on the course webpage), as well as “obvious” theorems of arithmetic and theorems from Gries and Schneider, to prove the stated implications.

8. $(P_0 \Rightarrow P_1 \land Q_0 \Rightarrow Q_1) \implies (\{P_1\} S \{Q_0\} \Rightarrow \{P_0\} S \{Q_1\})$

*Hint*: Assume the antecedant and prove the consequent.

9. If $\{x \geq y\} S \{x > 2y \land y > z\}$ and $\{y < 7\} S \{x > 2y \land y < z\}$, then $\{x > y \lor y \leq 4\} S \{x \geq 2y \land y \neq z\}$

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For the last two problems, identify both the weakest \( Y \) and the strongest \( Y \) satisfying the given “equation”. In developing your answers, it may be helpful to think in terms of *satisfying state sets* rather than predicates and to use these facts:

\[
\begin{align*}
    (P \land Q) &= \hat{P} \cap \hat{Q} \\
    (P \lor Q) &= \hat{P} \cup \hat{Q} \\
    [P \Rightarrow Q] &\equiv \hat{P} \subseteq \hat{Q}
\end{align*}
\]

As in class, where \( R \) is a predicate, \( \hat{R} \) refers to the set containing precisely those states that satisfy \( R \) (i.e., those states in which \( R \) evaluates to true).

10. \( Y : [P \land Y \Rightarrow P \land Q] \).

11. \( Y : [P \lor Q \Rightarrow P \lor Y] \).