Recall that if IF is the program \( \text{if } B_0 \rightarrow S_0 \mid \mid B_1 \rightarrow S_1 \text{ fi} \)

then \( \{ P \} \text{ IF } \{ Q \} \equiv \{ P \Rightarrow (B_0 \lor B_1) \} \land \{ P \land B_0 \} S_0 \{ Q \} \land \{ P \land B_1 \} S_1 \{ Q \} \)

which generalizes in the natural way when IF has more than two branches.

1. Prove (where \( p, q, \) and \( r \) are boolean variables)
   \[
   \{ r \} \text{ if } \neg p \rightarrow \text{skip} \quad \mid \quad p \rightarrow r := q \quad \text{fi} \\
   \{ r \equiv p \Rightarrow q \}
   \]

2. Prove
   \[
   \{ P : x > y \} \quad \text{if} \quad x > z \rightarrow x, z := z, x \quad \mid \quad y < z \rightarrow z, y := y, z \quad \text{fi} \\
   \{ Q : x \leq z \lor z \leq y \}
   \]

   Hint: By Contrapositive (Gries, 3.61), \( [P \Rightarrow B_0 \lor B_1] \) is equivalent to \( \neg(B_0 \lor B_1) \Rightarrow \neg P \)

3. Prove
   \[
   \{ P \land 0 \leq k < \#b \} \quad \text{if} \quad b.k \leq 0 \rightarrow \text{sum}, k := \text{sum} - b.k, k + 1 \quad \mid \quad b.k \geq 0 \rightarrow \text{sum} := \text{sum} + b.k; k := k + 1 \quad \text{fi} \\
   \{ P \}
   \]

where \( P : \text{sum} = (+i \mid 0 \leq i < k : |b.i|) \) and where \(|x|\) is the absolute value of \( x \), defined as follows:

\[
\[(|x| = x \equiv x \geq 0) \land (|x| = -x \equiv x \leq 0)\]

Notice that, in Problem 3, the two guarded commands (nested inside the selection command) are different in form, in that one is a multiple assignment and the other is a catenation. Hence, their proofs should not be mirror images of each other.

Notice that, in Problem 4, the program is a catenation of a selection command and an assignment command. Thus, to show that the Hoare Triple is valid you should make use of a catenation law to provide the appropriate postcondition for the selection command.