

SE 504 (Formal Methods and Models)
Spring 2024
HW #1: Predicate Strength/Weakness
Due: Thursday, Feb 1

For each of Problems 1 through 7, indicate the weakness/strength relationship that exists between the two given predicates, P and Q . Recall that there are four possibilities: P and Q are equivalent, P is strictly stronger than Q , P is strictly weaker than Q , or none of the above. For a more detailed treatment, follow the *On the Strength/Weakness Relationship between Predicates* link on the course web page.) For at least one problem, the theorems in *On Proofs Involving the Replacement of A by B , where A implies B* will be useful. Specifically, recall that weakening (respectively, strengthening) the antecedent of an implication strengthens (respectively, weakens) the implication. Meanwhile, weakening (respectively, strengthening) the consequent of an implication weakens (respectively, strengthens) the implication.

You must justify your answers, but you need not provide formal justifications for “obvious” theorems of arithmetic, such as $x > y \Rightarrow x \geq y$, or $x \geq y + 4 \Rightarrow x \geq y$, or $x < y \Rightarrow x \neq y$.

1. $P : x > 0 \wedge y > x - 1$ and $Q : x \geq 4 \wedge y \geq x$
2. $P : x > 0 \wedge y \geq x - 1$ and $Q : x \geq 4 \vee y > x$
3. $P : x > 1 \vee y < x$ and $Q : x > -5$
4. $P : x > 1 \wedge y < x$ and $Q : x > -5$
5. $P : x > -5 \wedge y < x$ and $Q : x = 0$
6. $P : x \geq 0 \Rightarrow y > z$ and $Q : x = 1 \Rightarrow y \geq z$
7. $P : f.k = 5$ and $Q : (\exists i \mid : f.i = 5)$

For the last two problems, identify both the weakest Y and the strongest Y satisfying the given “equation”. In developing your answers, it may be helpful to think in terms of *satisfying state sets* rather than predicates and to use these facts:

$$(\widehat{P \wedge Q}) = \hat{P} \cap \hat{Q}$$

$$(\widehat{P \vee Q}) = \hat{P} \cup \hat{Q}$$

$$[P \Rightarrow Q] \equiv \hat{P} \subseteq \hat{Q}$$

As in class, where R is a predicate, \hat{R} refers to the set containing precisely those states that satisfy R (i.e., in which R evaluates to true).

8. $Y : [P \wedge Y \Rightarrow P \wedge Q]$.
9. $Y : [P \vee Q \Rightarrow P \vee Y]$.